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Three-fluid heat exchangers with three thermal communications. Part B: effectiveness evaluation

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Abstract

Six different effectiveness parameters or figures of merit are defined based on the five identified engineering goals of three-fluid heat exchangers having three thermal communications. The effect of six non-dimensional design parameters on the defined figures of merit is discussed. It is shown that various effectivenesses of this class of heat exchangers strongly depend on the relative thermal capacities of three fluid streams. Any single definition is shown incapable of incorporating all of the five different objectives. A distinction is made between the thermal and temperature effectivenesses of heat exchangers.

1. Introduction

Effectiveness is a measure of the performance of any heat exchanger. In three-fluid heat exchangers with three thermal communications, considerable thermal energy is exchanged among all three streams simultaneously. Therefore, the heat exchanged between the cold and intermediate temperature streams or the hot and intermediate temperature streams cannot be neglected in comparison to the thermal energy exchanged between the cold and hot fluid streams. For the same reason, effectiveness definitions where thermal interaction of the hot and cold fluid streams with the intermediate temperature fluid stream are neglected will not give the true measure of the heating and cooling effectivenesses or the amount of actual energy exchanged among different streams.

Several effectiveness definitions have been proposed in the past to assess the performance of three-fluid heat exchangers. Most of these definitions give the temperature effectiveness of a particular stream and are defined as the ratio of the actual temperature difference to the maximum temperature difference that the stream of interest can attain [1–6]. Therefore, these definitions assess the performance of a three-fluid heat exchanger by its ability to achieve a maximum temperature difference for a selected stream. This is a valid approach to evaluate the performance of a heat exchanger. However, these definitions fail to give any measure of the actual amount of thermal energy being utilized inside the heat exchanger to accomplish a particular objective as a fraction of the total thermal energy supplied to the heat exchanger, which is also a very important indicator of the performance of any heat exchanger. Also, note that the temperature effectiveness definition is based on the actual temperature change of a single stream. Thus, at least three separate and different temperature definitions of this kind are possible for three-fluid heat exchangers.

Aulds and Barron [4] defined effectiveness for three-fluid heat exchangers with three thermal communications as the ratio of the actual heat transferred to the cold and intermediate fluids to the maximum heat that could be transferred to both of these streams. In their study, the configuration of the fluid streams was such that the cold and intermediate fluid streams were flowing in the same direction and the hot fluid stream was flowing in the counter direction to the other two (similar to our case P2). A specific objective of their heat exchanger has not been mentioned. However, given their definition the objective could be identified as to cool the

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Nomenclature

C_{12}	ratio of the thermal capacity of fluid 1 to 2, C_1/C_2
C_{32}	ratio of the thermal capacity of fluid 3 to 2, C_3/C_2
C_c	capacity of the cold fluid
C_h	capacity of the hot fluid
C_i	capacity of the intermediate inlet temperature fluid
NTU_1	number of transfer units based on the fluid 1, $(UA_1)/C_1$
R_1	ratio of the thermal resistance between fluids 1 and 2 and fluids 3 and 2, $(UA)_{32}/(UA)_{12}$
R_2	ratio of the thermal resistance between fluids 1 and 2 and fluids 3 and 1, $(UA)_{31}/(UA)_{12}$
$T_{c,in}$	inlet temperature of the cold fluid stream
$T_{c,out}$	outlet temperature of the cold fluid stream
$T_{h,in}$	inlet temperature of the hot fluid stream
$T_{h,out}$	outlet temperature of the hot fluid stream
$T_{i,in}$	inlet temperature of the intermediate temperature fluid stream
$T_{i,out}$	outlet temperature of the intermediate temperature fluid stream
$(UA)_{ch}$	overall conductance between the cold and hot fluid streams
$(UA)_{ci}$	overall conductance between the cold and intermediate temperature fluid streams
$(UA)_{hi}$	overall conductance between the hot and intermediate temperature fluid streams

Greek symbols

ε_c	heating thermal effectiveness of the cold fluid
ε_h	cooling thermal effectiveness of the hot fluid
$\varepsilon_{i,h}$	heating thermal effectiveness of the intermediate temperature fluid
$\varepsilon_{i,c}$	cooling thermal effectiveness of the intermediate temperature fluid
ν_c	heating temperature effectiveness of the cold fluid
ν_h	cooling temperature effectiveness of the hot fluid
$\nu_{i,h}$	heating temperature effectiveness of the intermediate temperature fluid
$\nu_{i,c}$	cooling temperature effectiveness of the intermediate temperature fluid
$\theta_{3,in}$	non-dimensional inlet temperature of the third fluid, $(T_{3,in} - T_{1,in})/(T_{2,in} - T_{1,in})$

Subscripts

1	hot fluid
2	cold fluid
3	intermediate temperature fluid
c	cold fluid
h	hot fluid
i	intermediate temperature fluid stream
in	position where fluid enters the heat exchanger
out	position where fluid leaves the heat exchanger

hot fluid. It can be seen from their effectiveness definition that it would change if the objective changes (i.e., to heat the cold fluid). Also, only two possible operational conditions have been considered: one, when C_c and C_i are both less than C_h and the second, when C_h is less than both C_i and C_c.

Sekulic and Kmecko [7] realized that the definition of the effectiveness depends on the objective of the heat exchanger. They have also realized that a thorough analysis of the effectiveness of the three-fluid heat exchangers does not exist. In their words, "It is interesting to note, however, that a thorough analysis of the concept of the three-fluid heat exchanger effectiveness does not exist". Therefore, to fulfil this gap and provide a basis for further studies of a thermal performance figure of merit for three-fluid heat exchangers, they have studied the case of three-fluid heat exchangers with only two thermal communications. Their objective has been explicitly stated as to maximize the enthalpy change of the middle stream in a three-fluid heat exchanger with two thermal communications.

It can be concluded from the above discussion that no single definition is currently available to the knowledge of authors to evaluate the general performance of three-fluid heat exchangers with three thermal communications. Therefore the goals of this work are to provide a thorough analysis of and develop expressions for various figures of merit for this special class of heat exchangers based on their objectives.

2. Effectiveness

Different effectiveness parameters are developed for three-fluid heat exchangers based on the specific objectives of these heat exchangers. These definitions are general and can also be applied to two-fluid heat exchangers. Five different objectives of three-fluid heat exchangers may be identified, including, (1) heating the cold fluid, (2) cooling the hot fluid, (3) cooling the intermediate fluid, (4) heating the intermediate fluid, and (5) maximizing the enthalpy change of the central fluid stream or the two lateral fluid streams. These objectives encompass all the objectives mentioned previously [4–7]. Also the effectiveness expressions for assessing the performance should have the following properties [6].

1. The figure of merit represents the measure of performance with respect to the desired engineering task.
2. The figure of merit is expressed in dimensionless form.
3. The figure of merit has a range between 0 and 1.

Heating effectiveness of a cold fluid for any three-fluid heat exchanger can be defined based either on its temperature effectiveness or its ability to capture thermal energy from the other two fluid streams. Heating temperature effectiveness v_c is defined as the actual difference between the cold outlet and inlet temperatures to the maximum possible temperature difference that this stream can attain.

$$v_c = \frac{T_{c,out} - T_{c,in}}{T_{h,in} - T_{c,in}} \tag{1}$$

In the traditional sense, where effectiveness of any heat exchanger is defined as the actual heat transfer to the maximum possible heat transfer, heating thermal effectiveness of the cold fluid ε_c can be defined as

$$\varepsilon_c = \frac{Q_{actual}}{Q_{max}} \tag{2}$$

where,

$$Q_{actual} = C_c(T_{c,out} - T_{c,in}) \tag{3}$$

The expression for Q_{max} for the cold fluid depends on the relative magnitudes of the thermal capacities of the different streams of the three-fluid heat exchanger. It should also be noted that the cold fluid should flow opposite to the other two fluid streams if maximum heat transfer to the cold fluid from the other two streams is desired. In the case where the thermal capacity of the cold fluid is higher than the thermal capacities of the other two fluids, and the cold fluid is flowing counter to the other two streams, Q_{max} is represented by

$$Q_{max} = C_h(T_{h,in} - T_{c,in}) + C_i(T_{i,in} - T_{c,in}) \tag{4}$$

For all other combinations of thermal capacities of the three fluids, maximum heat transfer to the cold fluid is represented by

$$Q_{max} = C_c(T_{h,in} - T_{c,in}) \tag{5}$$

This conclusion about the maximum heat transfer to the cold fluid for all other possible combinations of thermal capacities is made because, if the objective of the heat exchanger is only to heat a cold fluid to its maximum, the resistance between the cold and intermediate temperature fluid streams and hot and intermediate temperature fluid streams can always be made infinite. Ideally, if the intermediate fluid is present and the cold fluid capacity is smaller than the thermal capacities of the other two fluids, the outlet temperature of the cold fluid will approach the temperature $T_{c,out}$ [4], expressed as

$$T_{c,out} = \left[\frac{(UA)_{ch}}{(UA)_{ci}} T_{h,in} + T_{i,in}\right]\left[\frac{(UA)_{ch}}{(UA)_{ci}} + 1\right]^{-1} \tag{6}$$

From Eq. (6) it is evident that $T_{c,out}$ will attain the maximum possible temperature, $T_{h,in}$ only when the conductance between the cold and intermediate temperature fluid streams is zero. Similarly, it can also be shown that maximum heat transfer to the cold fluid will be represented by Eq. (5) even if the thermal capacity of the cold fluid is bracketed by the thermal capacities of the hot and intermediate temperature fluids.

Cooling effectiveness of a hot fluid for any three-fluid heat exchanger can be defined based either on its temperature effectiveness or its ability to release thermal energy to the other two fluid streams. Cooling temperature effectiveness v_h is defined as the actual difference between the hot inlet and outlet temperatures to the maximum possible temperature difference that this stream can attain.

$$v_h = \frac{T_{h,in} - T_{h,out}}{T_{h,in} - T_{c,in}} \tag{7}$$

In the traditional sense, cooling thermal effectiveness of the hot fluid $\varepsilon_h$can be defined as

$$\varepsilon_h = \frac{Q_{actual}}{Q_{max}} \tag{8}$$

where,

$$Q_{actual} = C_h(T_{h,in} - T_{h,out}) \tag{9}$$

In the case when the thermal capacity of the hot fluid is greater than the individual thermal capacities of the other two fluids, and the hot fluid is flowing counter to the other two streams, Q_{max} is given by

$$Q_{max} = C_c(T_{h,in} - T_{c,in}) + C_i(T_{h,in} - T_{i,in}) \tag{10}$$

For all other combinations of thermal capacities of the three fluids, maximum heat transfer from the hot fluid is represented by

$$Q_{max} = C_h(T_{h,in} - T_{c,in}) \tag{11}$$

Ideally, if the intermediate temperature fluid is present and the hot fluid capacity is smaller than both the thermal capacities of the other two fluids, the outlet temperature of the hot fluid should approach the temperature $T_{h,out}$ which may be expressed as [4]

$$T_{h,out} = \left[\frac{(UA)_{ch}}{(UA)_{hi}} T_{c,in} + T_{i,in}\right]\left[\frac{(UA)_{ch}}{(UA)_{hi}} + 1\right]^{-1} \tag{12}$$

Heating or cooling effectiveness of an intermediate temperature fluid inside any three-fluid heat exchanger can also be defined based either on its temperature effectiveness or its ability to capture or lose thermal energy from the other two fluid streams.

Temperature effectiveness is defined as the actual difference between the outlet and inlet temperatures of the intermediate temperature fluid stream to the maximum possible temperature difference that this stream can attain

$$\nu_{i,h} = \frac{T_{i,out} - T_{i,in}}{T_{h,in} - T_{i,in}} \tag{13}$$

$$\nu_{i,c} = \frac{T_{i,in} - T_{i,out}}{T_{i,in} - T_{c,in}} \tag{14}$$

Eqs. (13) and (14) represent the temperature heating effectiveness $\nu_{i,h}$ and temperature cooling effectiveness $\nu_{i,c}$ of the intermediate temperature fluid stream, respectively.

In the traditional sense, heating thermal effectiveness of the intermediate temperature fluid $\varepsilon_{i,h}$ can be defined as

$$\varepsilon_{i,h} = \frac{Q_{actual}}{Q_{max}} \tag{15}$$

where,

$$Q_{actual} = C_i(T_{i,out} - T_{i,in}) \tag{16}$$

Similarly, thermal cooling effectiveness of the intermediate temperature fluid stream $\varepsilon_{i,c}$ is defined as

$$\varepsilon_{i,c} = \frac{Q_{actual}}{Q_{max}} \tag{17}$$

where,

$$Q_{actual} = C_i(T_{i,in} - T_{i,out}) \tag{18}$$

For any combination of thermal capacities, the maximum heat transfer Q_{max} for the intermediate temperature fluid stream is given by Eqs. (19) and (20) for heating and cooling, respectively.

$$Q_{max} = C_i(T_{h,in} - T_{i,in}) \quad \text{(intermediate fluid heating)} \tag{19}$$

$$Q_{max} = C_i(T_{i,in} - T_{c,in}) \quad \text{(intermediate fluid cooling)} \tag{20}$$

Notice that if Eqs. (19) and (20) are used for the maximum heat transfer for heating and cooling of the intermediate temperature fluid, Eqs. (13) and (14) will be the same as Eqs. (15) and (17). Ideally, if all the fluids are present and the capacity of the intermediate temperature fluid is smaller than the thermal capacities of the other two fluids, the outlet temperature of this fluid should approach the intermediate outlet temperature $T_{i,out}$ given as

$$T_{i,out} = \left[\frac{(UA)_{hi}}{(UA)_{ci}} T_{h,in} + T_{c,in}\right]\left[\frac{(UA)_{hi}}{(UA)_{ci}} + 1\right]^{-1} \tag{21}$$

From Eq. (21), $T_{i,out}$ will attain the maximum possible temperature $T_{h,in}$ only when the overall conductance between the cold and intermediate temperature fluid streams is zero. In the same way, $T_{i,out}$ will attain the minimum possible temperature $T_{c,in}$ only when the overall conductance between the hot and intermediate temperature fluid streams is zero.

It should be noted at this point that in practice, all two-fluid heat exchangers operate with atmosphere around them. However, the effectiveness of the ideal two-fluid heat exchangers is defined assuming that there exists no heat source or sink inside the heat exchanger and that there is no thermal communication between the heat exchanger and ambient. These facts suggest that any expression for the effectiveness of a three-fluid heat exchanger should reduce to the two-fluid heat exchanger effectiveness expression when the thermal capacity of one of the streams (i.e., the stream representing the ambient) is zero and the thermal resistance between the ambient and the other two streams is assumed to be infinite. It can be noticed from Eqs. (2) and (8) that they will reduce to the standard two fluid effectiveness expression. In order for this simplification to occur, it is assumed that the first, second and third streams represent cold, hot and intermediate temperature fluid streams, respectively, the third fluid represents the ambient (with zero thermal capacity), and the heat transfer resistance of this fluid with the other two is infinite.

It should also be noted that all of the above effectiveness equations cannot be defined in terms of the general non-dimensional temperatures $\theta_j s$ [8] as the effectiveness relationships are based on knowledge of hot, cold and intermediate fluid streams. In the derivation of θ_j for the four flow arrangements, any of the streams can be hot, cold or intermediate without losing the generality of the computed temperature profiles [8]. However, in defining expressions for different effectivenesses, it is necessary to know which fluid stream is hot, cold or intermediate. This constraint on these expressions does not make these definitions less general, as in practice, one will know which fluid stream is cold, hot or intermediate. Since temperature profiles can be determined from the expressions derived by Shrivastava and Ameel [8], effectiveness of the heat exchanger can be evaluated easily based on the objective of the heat exchanger by using any of the several effectiveness equations presented here.

3. Results and discussion

Different effectiveness parameters have been developed in the previous section for three-fluid heat exchangers with three or fewer thermal communications. It has been clearly stated that to assess the effectiveness of this class of heat exchangers the objective of the heat exchanger should be known. It should also be known which fluid has the hot, cold, or intermediate inlet temperature. It is assumed in this section that the first, second, and third fluids are the cold, hot, and intermediate temperature fluids, respectively. Also, it has been shown [8] that cases P2, P3, and P4 are actually similar in the case of three-fluid heat exchangers with three thermal communications. Fig. 1 shows that the trends of the different effectiveness curves in cases P1 and P2 are similar. Since the values for P2 are always greater than P1, from now on only the effect of different design parameters on the effectiveness of case P2 (the more effective of the two) for various objectives is presented. Also, in studying the effect of a particular design parameter, the rest of the design parameters are fixed to arbitrary values.

The effect of R_1 on the different effectiveness parameters is shown in Fig. 1 for cases P1 and P2. R_1 is defined as the ratio of the overall thermal resistance between fluids 1 and 2 to that between fluids 3 and 2. Therefore, an increase in the value of R_1 can be interpreted as a relative decrease in the overall thermal resistance between fluids 3 and 2 in comparison to the overall thermal resistance between fluids 1 and 2 or a relative increase in the overall thermal resistance between fluids 1 and 2 in comparison to that between fluids 3 and 2, or both. However, as R_2 (defined as the ratio of the overall thermal resistance between fluids 1 and 2 to that between fluids 3 and 1) is fixed, an increase in R_1 should only be identified with the relative decrease in the overall thermal resistance between fluids 3 and 2 in comparison to that between fluids 3 and 1.

As the overall thermal resistance between fluids 3 and 2 decreases relatively with the increase in R_1 the relative thermal interaction between these two streams increases. This leads to a rise in the outlet temperature of fluid 3. Thus, heating thermal effectiveness for the intermediate temperature fluid stream $\varepsilon_{i,h}$ increases with the increase in R_1. The curve for the cooling thermal effectiveness of the intermediate temperature fluid stream $\varepsilon_{i,c}$ is not shown as it does not exist when the outlet temperature of fluid 3 is more than the inlet temperature of fluid 3. Also, due to the relatively enhanced thermal interaction between hot and intermediate temperature fluids, relatively more heat is lost from the hot fluid to the intermediate temperature fluid as R_1 increases. This leads to a relatively lower outlet temperature for the hot fluid stream. Thus, as R_1 increases, cooling temperature and thermal effectivenesses of the hot fluid, ν_h and ε_h increase.

As the overall thermal resistance between fluids 3 and 1 is lower than the overall thermal resistance between fluids 1 and 2, there will be more thermal interaction between fluids 1 and 3 than between fluids 1 and 2. Therefore, fluid 1 will be more affected by the temperature distribution of fluid 3 than the temperature distribution of fluid 2. As fluid 3 has a relatively higher temperature distribution with the increase in R_1, this

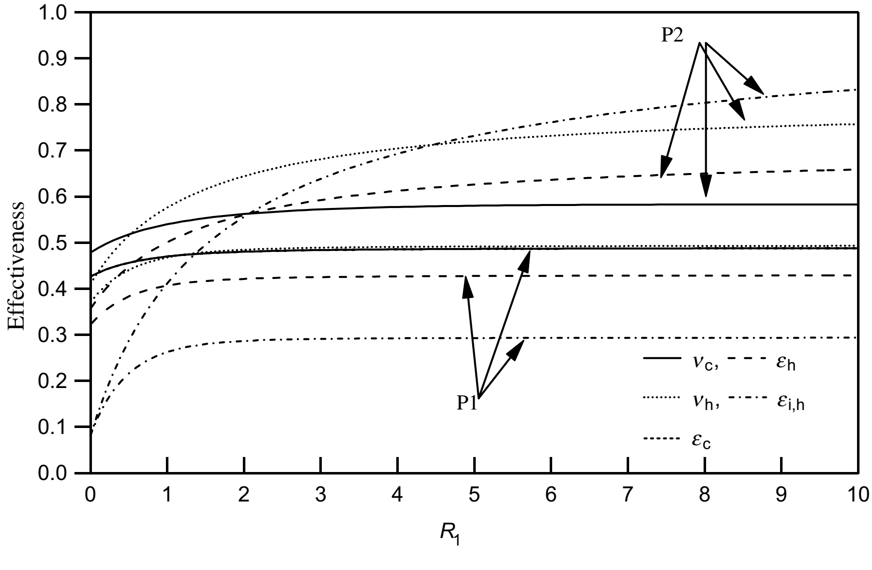

Fig. 1. Effect of R_1 on various effectiveness parameters for cases P1 and P2 for $R_2 = 1.5$, $C_{12} = 0.8$, $C_{32} = 0.5$, $\mathrm{NTU}_1 = 1.0$, and $\theta_{3,\mathrm{in}} = 0.3$.

leads to a relative rise in the temperature distribution of fluid 1. Therefore, heating temperature and thermal effectivenesses of the cold fluid, ν_c and ε_c increase with the increase in R_1 (Fig. 1). The effects of R_2 and NTU_1 on the various effectiveness parameters for case P2 are shown in Figs. 2 and 3, respectively. These effects may also be explained using arguments similar to those used with Fig. 1.

The effect of C_{12} on the different effectiveness parameters is shown in Fig. 4. C_{12} is defined as the ratio of the thermal capacity of fluid 1 to that of fluid 2. Therefore as the value of C_{12} is increased, either the thermal capacity of fluid 1 is increased relative to fluid 2 or the thermal capacity of fluid 2 is decreased relative to the first, or both. C_{32} is defined as the ratio of the thermal capacity of fluid 3 to that of fluid 2. If C_{32} is constant, an increase in C_{12} should only be interpreted as an increase in the thermal capacity of fluid 1.

With the increase in C_{12} the relative thermal capacity of the cold fluid is increased, producing a decrease in the outlet temperature of the hot fluid. Therefore, as C_{12} increases, the cooling temperature effectiveness of the second fluid (hot fluid) ν_h increases.

For all cases in which the capacity of the second fluid is the largest (i.e., $C_{12} < 1$, and $C_{32} < 1$), cooling thermal effectiveness of fluid 2 (hot fluid) ε_h is directly propor-

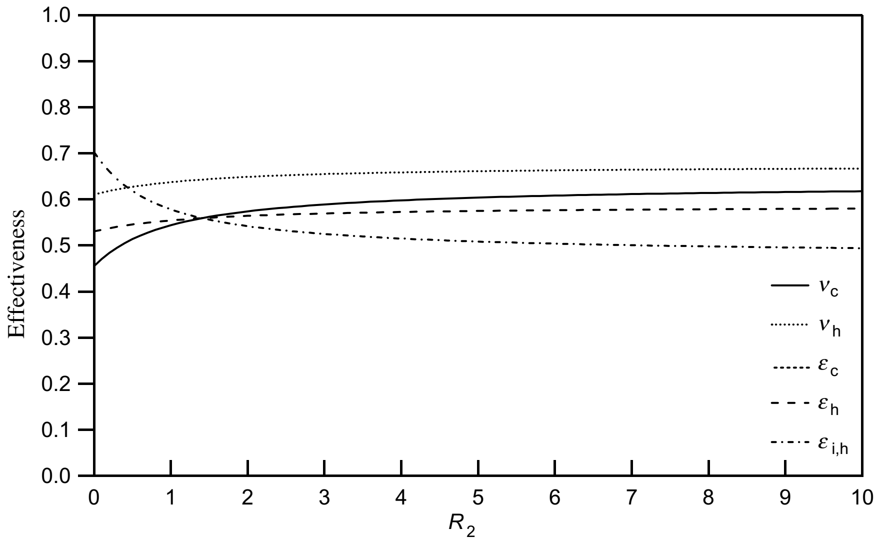

Fig. 2. Effect of R_2 on various effectiveness parameters for case P2 for $R_1 = 2.0$, $C_{12} = 0.8$, $C_{32} = 0.5$, $NTU_1 = 1.0$, and $\theta_{3,in} = 0.3$.

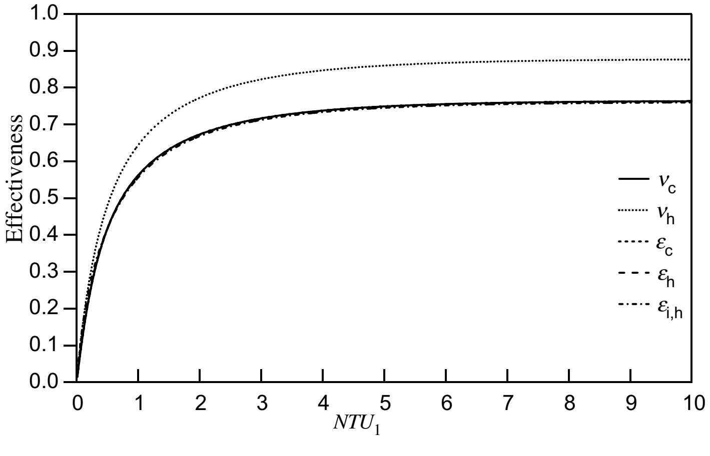

Fig. 3. Effect of NTU_1 on various effectiveness parameters for case P2 for $R_1 = 2.0$, $R_2 = 1.5$, $C_{12} = 0.8$, $C_{32} = 0.5$, and $\theta_{3,in} = 0.3$.

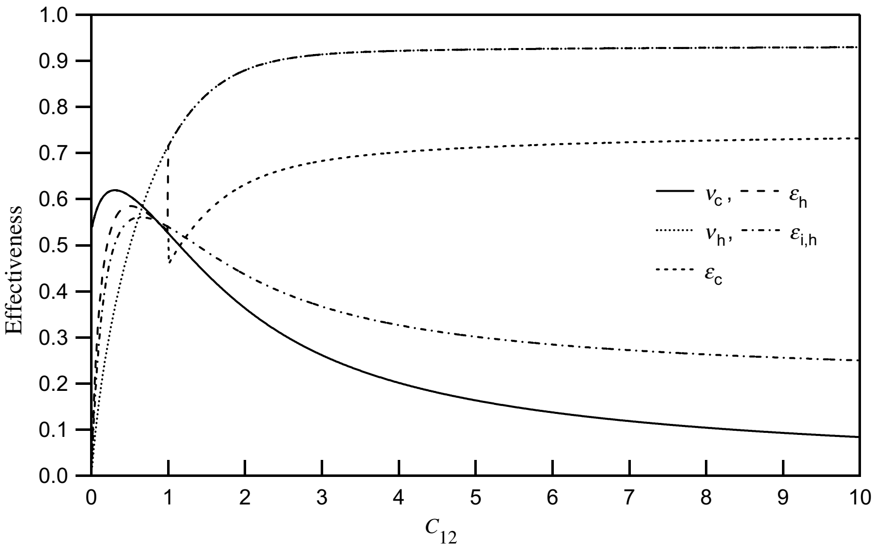

Fig. 4. Effect of C_{12} on various effectiveness parameters for case P2 for $R_1 = 2.0$, $R_2 = 1.5$, $C_{32} = 0.5$, $NTU_1 = 1.0$, and $\theta_{3,in} = 0.3$.

tional to $(1 - \theta_{2,out})$ and inversely proportional to C_{12}. For very small values of C_{12}, $(1 - \theta_{2,out})$ grows faster than C_{12} itself. Therefore, cooling thermal effectiveness ε_h increases initially. This effect can be attributed to the combined interaction of all the three fluid streams for the given design parameters. Later ε_h decreases with the increase in C_{12}. For all other cases $(C_{12} \geqslant 1)$ cooling thermal effectiveness of fluid 2 (hot fluid) ε_h is directly proportional to $(1 - \theta_{2,out})$ only. Therefore, when $C_{12} \geqslant 1$, fluid 1, instead of fluid 2, becomes the most important fluid (i.e., $C_1 > C_2$). Thus as C_{12} is increased to values $\geqslant 1$, the increasing thermal capacity of fluid 1 will continue decreasing the value of $\theta_{2,out}$. This will result in a continuous increase of the cooling thermal effectiveness of fluid 2. The sudden jump in the value of cooling thermal effectiveness of the hot fluid ε_h when $C_{12} = 1$ can be attributed to the fact that, at this value of C_{12}, fluid 1 instead of fluid 2 governs the maximum heat transfer attainable.

Heating thermal effectiveness of the intermediate temperature stream $\varepsilon_{i,h}$ at low C_{12}, corresponds to an initial increase in the outlet temperature of fluid 3 with increasing C_{12}. This results in an increase in the heating thermal effectiveness of the intermediate temperature stream for low C_{12}, as shown in Fig. 4. This early rise in the value of $\varepsilon_{i,h}$ for low values of C_{12} can be attributed to the combined interaction of all the three streams for the assigned values of the design parameters. As the value of C_{12} is increased further, increasing thermal capacity of fluid 1 forces fluid 3 to cool more. This leads to a decrease in $\varepsilon_{i,h}$ and an increase in $\varepsilon_{i,c}$ respectively (Fig. 4). According to the definitions, heating/cooling thermal effectivenesses of the intermediate temperature fluid stream are not defined when the outlet temperature of the third stream takes lower/higher values than the inlet temperature of this stream, respectively. This definition produces values between 0 and 1 for the effectivenesses $\varepsilon_{i,h}$ and $\varepsilon_{i,c}$ which is consistent with the standard procedure of defining effectiveness. Thus no curve for the cooling thermal effectiveness of the intermediate temperature fluid stream $\varepsilon_{i,c}$ is shown in Fig. 4.

Again the thermal interaction among all the three streams for low values of C_{12} for the given values of design parameters results in a relative increase in the output temperature of fluid 1 $\theta_{1,out}$ as C_{12} increases (Fig. 4). For all cases, except when $(C_{12} > C_{32})$ and $(C_{12} > 1)$, $\varepsilon_c = \theta_{1,out}$ Therefore, for low values of C_{12}, the heating thermal effectiveness of the cold fluid ε_c increases. Later, as the thermal capacity of fluid 1 increases further, the outlet temperature of fluid 1 decreases. This results in a decreasing heating temperature effectiveness ν_c for this class of heat exchangers. As the heating temperature effectiveness is the same as the heating thermal effectiveness for the cases when thermal capacity of the first fluid is not the largest (when the coupled condition $C_{12} > C_{32}$ and $C_{12} > 1$ is not true), heating thermal effectiveness of the cold fluid ε_c also decreases and both curves show the same behavior. When the thermal capacity of the first fluid is the largest of all ($C_{12} > C_{32}$ and $C_{12} > 1$), heating thermal effectiveness ε_c is proportional to the product of C_{12} and $\theta_{1,out}$ which increases with the increase in C_{12} even when $\theta_{1,out}$ is decreasing as the product of the two increases.

The apparent discontinuity in ε_h and ε_c at $C_{12} = 1$ is actually a result of the definitions of the two effectiveness parameters which take on two values dependent on the maximum possible heat transfer. The maximum possible heat transfer has previously been indicated to change at $C_{12} = 1$.

The effect of C_{32} on different effectiveness parameters for case P2 is shown in Fig. 5. The changes that occur with C_{32} may be explained using arguments similar to those used with Fig. 4.

The effect of $\theta_{3,\mathrm{in}}$ on the different effectiveness parameters for case P2 is shown in Fig. 6. As the value of $\theta_{3,\mathrm{in}}$ is increased from 0 to 1, the amount of heat present in the system, increases. Therefore, an increase in $\theta_{3,\mathrm{in}}$ leads to an increase in the heating temperature and thermal effectivenesses of the cold fluid v_c and ε_c and a decrease in the cooling temperature and thermal effectivenesses of the hot fluid v_h and ε_h As more energy is available to the third fluid, more heat is lost from the third fluid to the other two fluid streams. This leads to a decrease and increase in the heating and cooling thermal effectiveness of the third fluid, respectively. Note that $\varepsilon_{i,h}$ is undefined for $\theta_{3,\mathrm{in}} > 0.736$ and $\varepsilon_{i,c}$ is undefined for

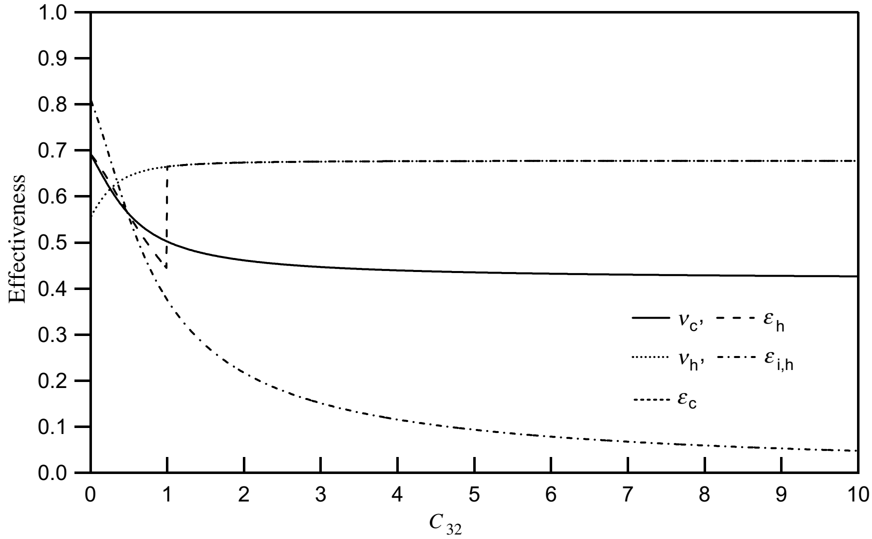

Fig. 5. Effect of C_{32} on various effectiveness parameters for case P2 for $R_1 = 2.0$, $R_2 = 1.5$, $C_{12} = 0.8$, $\mathrm{NTU}_1 = 1.0$, and $\theta_{3,\mathrm{in}} = 0.3$.

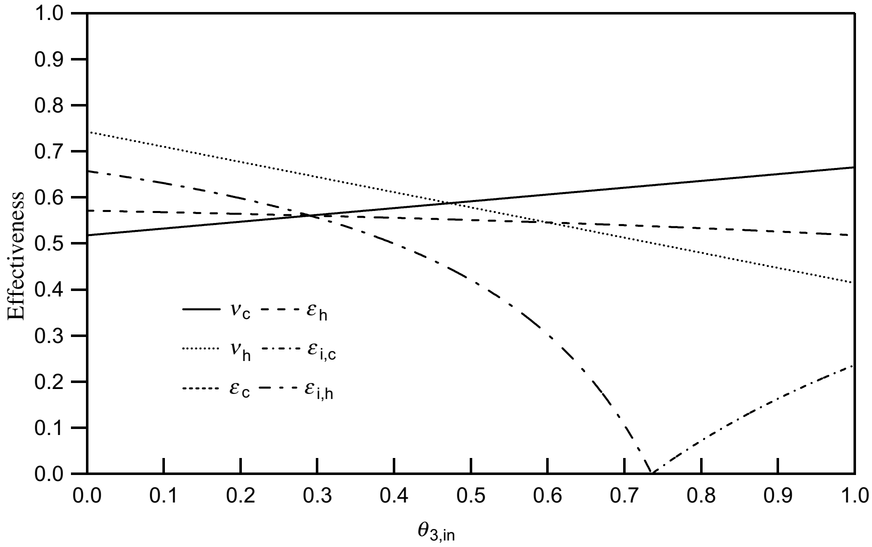

Fig. 6. Effect of $\theta_{3,\mathrm{in}}$ on various effectiveness parameters for case P2 for $R_1 = 2.0$, $R_2 = 1.5$, $C_{12} = 0.8$, $C_{32} = 0.5$, and $\mathrm{NTU}_1 = 1.0$.

$\theta_{3,in} < 0.736$ for the specific conditions used in Fig. 6. $\theta_{3,in} = 0.736$ represents a condition that will produce an outlet temperature equal to the inlet temperature of fluid 3 for the set of design parameters chosen. All of the trends exhibited in Fig. 6 for case P2 also occur for a P1 case.

4. Conclusions

Five different objectives, and therefore six different figures of merit, are identified for the first time in the present work for three-fluid heat exchangers with three thermal communications. The effect of the six previously identified [8,9] design parameters on different effectiveness parameters is studied and explained. It is shown that in the case of three-fluid heat exchangers with three thermal communications, the effectiveness of the heat exchanger depends on the arrangements of the fluids, their flow configurations, and the objective of the heat exchanger. Unlike two-fluid heat exchangers, a single general definition of effectiveness is not possible due to their, many times, conflicting requirements. Therefore, several different objective specific definitions are proposed to evaluate the performance (effectiveness) of this class of heat exchangers.

In general, model P2 performs better than model P1 for all the specified objectives. High values of R_1 and NTU_1 are found to increase the effectiveness of three-fluid heat exchangers in all respects. Similarly, high values of R_2 are found to increase all the effectivenesses of this class of heat exchangers except $\varepsilon_{i,h}$. High values of C_{12} are found to have an adverse effect on $\varepsilon_{i,h}$ and v_c. High values of C_{32} have a negative effect on $\varepsilon_{i,h}$, ε_c and v_c High values of $\theta_{3,in}$are found to enhance all the effectivenesses of three-fluid heat exchangers except $\varepsilon_{i,h}$ and v_h.

Some effectiveness parameters are discontinuous at $C_{12} = 1$. This kind of singularity predicted for a few effectiveness parameters at $C_{12} = 1.0$ is a physical possibility because in the two cases when $C_{12} < 1$ or $C_{12} > 1$ the maximum attainable heat transfer is different. Also, this change in maximum attainable heat transfer is abrupt at $C_{12} = 1.0$. Therefore, the possibility of a discontinuity is inherent in the definition of these effectiveness parameters. Again, different effectiveness definitions proposed will facilitate more effective heat exchanger design. Obviously it would be convenient to have a single general definition of effectiveness. However, since there are multiple objectives for this class of heat exchangers, a single effectiveness definition is not found sufficient to convey all the possible performance information.

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